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# The effects of the tax system on education decisions and welfare<sup>\*</sup>

Lari A. Viiano<sup>\*\*</sup>

## Abstract

I study the effects of a linear tax schedule on educational decisions and welfare in a two period model where the educational decision is discrete and its return is uncertain. I find that a linear tax rate has a positive effect on the number of agents who decide to acquire higher education. This effect becomes negative when the revenue is returned as a lump-sum transfer. The government can influence the education level to some extent using a linear tax schedule. However this policy always has a negative welfare effect. I also find that if there is a revenue need a welfare maximizing government will prefer a linear tax to a lump-sum tax.

**Keywords:** Education, taxation, welfare.

**JEL Classification:** I20,I30,H20,H31

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# 1 Introduction.

Governments impose income taxes. Reasons might be several, a simple revenue need, provision of public goods, to finance public provision of private goods (education, sanity...), redistributive purpose. Anyhow, income taxes are used by nearly all governments.

Without entering into the reasons that drive a government to impose a income tax schedule, nor in deep in the destination of such revenue, these taxes produce several effects on the society. Probably the most important and more studied tax effect is the effect on labor supply decisions, and the induced changes in consumption, welfare and production. The literature on optimal taxation deals with redistribution, labor supply and welfare, but the results are controversial in most cases. In his seminal approach, Mirrlees (1971) obtains an optimal tax schedule with a decreasing marginal tax rate that is zero for the most able agents.

However these are not the only effect of taxes. Educational decisions are also affected. The main return of education is an expected increase in future wages and one of the most important components of the cost of education are the forgone earnings. Both returns and costs of education are then clearly affected by the prevailing tax schedule.

The relation between taxes and the educational choice has been studied extensively under two main approaches: human capital accumulation theories and screening or signaling theories. In both approaches it is common to assume an inelastic labor supply, so that utility depends only on consumption, or to assume a separable utility function. The main conclusion in both approaches is a negative or neutral effect of taxes on education. The effect is negative when non-deductible costs exist, and it is neutral in the absence of them.

Still taxes might have positive effects on the educational choice of agents. The inclusion of the labor supply as a choice variable in the model can induce positive effects on the educational choice, as shown in Viianito(2007), or when the return to education is uncertain, Poutvaara(2002).

The aim of this paper is to analyze jointly the effect on labor supply and the educational decision, under uncertainty. Endogenize the educational decision and the labor supply choice when the return to education is uncertain, and realize a meaningful welfare analysis, comparing the performance of a linear tax schedule with respect to a lump sum

tax.

The educational decision and the labor supply decision are clearly and deeply related. The return to education is not merely a higher consumption due to a higher wage, it is a completely different restriction in the consumption-leisure choice. Agents will derive utility from both consumption and leisure choosing their labor supply endogenously. This includes an additional dimension to the problem. I will use a utility function that includes the nature of the relation between consumption and leisure, concretely a CES type function.

In the model, education is treated as a binomial choice: To acquire or not a predetermined level of education that is related to a predetermined wage structure. The cost of education depends on the innate ability of agents, more able agents spend less time obtaining education. There will exist a threshold level of ability that differentiates those agents who educate from those who do not, being the most able agents those who choose to acquire education, since their cost is lower.

The wage structure that agents will face is fixed and depends only on their educational choice. Agents with the same educational level must face the same wage structure.

The model includes uncertainty in the return of education as it is of paramount importance in the educational decision. Education is a long-term investment in which agents incur in an immediate cost in order to obtain a future benefit in the form of a higher expected wage. As Levhari and Weiss (1974) argued, there are two main sources of uncertainty: uncertainty in inputs (quality of schooling, dropout...) and uncertainty in outputs (imperfect knowledge of future market conditions...). This uncertainty is not insurable nor diversifiable and it is crucial when agents decide to become educated or not. Of course, in the same way, to remain uneducated is not exempt of uncertainty, but this level of uncertainty can be used as a reference point for the uncertainty related to education. Is then education more or less uncertain than remaining uneducated? There exists a wide discussion about the nature and importance of the uncertainty related to education. There are arguments in both directions.<sup>1</sup> In this paper, and following theoretical works as Levhari and Weiss (1974), Eaton and Rosen (1980b), Hamilton (1987), Poutvaara (2002) or empirical ones as Hartog, Oosterbeek and Teulings (1993) or Chen (2002), I assume that education increases uncertainty. In particular I assume that education is uncertain in outputs in the following way: educated agents have access

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<sup>1</sup>See Sebastian Buhai (2003).

to a wider set of jobs than uneducated agents. The wage structure that educated agents face is wider and therefore the variance of possible wages is higher. To simplify the analysis I will assume that uneducated agents face a fixed wage while educated agents face with equal probability a high or a low wage, being the low wage as least as much as the uneducated wage. Observe that the increase in wage uncertainty does not imply risk in terms of wage since both educated wages are weakly higher than the uneducated wage.

To summarize I have a two period model where agents derive utility only from the second period consumption and leisure.<sup>2</sup> In the first period agents supply labor inelastically at a common wage and save all income to be consumed in the second period. In the first period agents can choose to educate in order to obtain a higher expected wage in the second period. Education requires to invest time and money. The time cost depends on ability and it is measured in terms of lost income. In the second period, after the wage realization, agents choose their labor supply and consumption in order to maximize their utility function. The model is tested in several scenarios including a comparison between a linear and a lump-sum tax.

This paper has several results. The most interesting one is that, in the absence of direct costs of education, taxation is positive for education. This relation is due to the complementarity between consumption and leisure and it does not depend on the uncertainty related to education. In fact uncertainty only harms education if it is seen as a mean-preserving spread of the expected wage. More agents will choose to educate if the educated wage structure offers the expected wage with certainty.<sup>3</sup>

Transfers have a negative effect on education, as expected. If all tax revenue is returned as transfers, the effect on education is negative. Due to the negative effect of a full transfer policy and to the positive effect of taxation, there is always room to affect education decisions with an appropriate tax-transfer combination. There always exists a feasible tax-transfer combination that keeps education constant.

Tuition costs have also a negative effect on education. This effect increases with taxes and it is strong enough to easily counteract the positive effect of taxes when education is free.

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<sup>2</sup>See Viiano(2007) for a complete explanation of the period choice.

<sup>3</sup>This implies risk aversion with respect to wages.

When the revenue is used to finance education there are two positive effects, one related to the reduction of tuition costs and the positive effect related to taxes. Once education is fully subsidized, previous results apply and any further increase in taxes will increase education. If the additional revenue is returned as transfers, the positive effect on education diminishes and it might turn negative if transfers are high enough.

In all these scenarios taxes have a negative effect on social welfare when education has no externalities. Still the use of a lump-sum tax, to cover a revenue need, produces more distortion and a higher welfare loss than a linear tax rate collecting the same amount of revenue.

The rest of the paper is organized as follows: Section 2 describes the linear tax model, Section 3 provides results for the linear tax model and a comparison with a lump-sum tax and Section 4 concludes.

## 2 Model

I propose a model where a continuum of heterogeneous agents live for two periods of length one. Agents differ in their innate ability to acquire education ( $h$ ) that is distributed according to some distribution function  $F(h)$  on  $[0, 1]$ .

In the first period agents decide whether to acquire education or not. If they decide to do so they must invest an amount of time  $(1 - h)$ . Education might also require to pay a monetary cost  $T$ , that may differ from the real cost of education  $Q$  due to the existence of a governmental subsidy  $\xi$ , thus  $T = Q - \xi$ . Agents do not derive any utility from consumption or leisure in the first period. The time endowment in the first period is exhausted working and becoming educated and all the labor income is saved at some interest rate  $r$  to be consumed in the second period. In the first period all agents face a common wage  $w_0$ .

In the second period the uneducated agents will face a wage  $w_U$ , whereas the educated agents will face, with probability  $p$ , a high wage  $w_H$  or, with probability  $(1 - p)$ , a low wage  $w_L$ , so that  $w_0 \leq w_U \leq w_L < w_H$ . Once agents observe their wage realization they choose their labor supply and consumption to maximize their utility.

## 2.1 The utility function

I assume that preferences are represented by the following CES utility function:

$$U(C, L) = (\alpha C^\rho + \beta L^\rho)^{1/\rho}, \quad (1)$$

where  $C$  is consumption and  $L$  is leisure, with  $L \in [0, 1]$ ,  $\alpha, \beta > 0$  and  $\rho < 0$ . A negative  $\rho$  implies complementarity between  $C$  and  $L$ . This function is quite general, it has income effects, it is backward sloping in leisure when the wage is high enough and it has been found empirically to fit quite well actual labor supply decisions.

Stern (1976) has parameterized this utility function to fit actual labor supply decisions in the U.S. He estimates a value of  $\rho \simeq -1.5$  and, setting  $\beta = (1 - \alpha)$ , a value for  $\alpha \simeq 0.99$ . I will use similar values in the simulations below.<sup>4</sup>

## 2.2 The budget constraint

Agents face different budget constraints in the second period according to their ability and their education decisions. Furthermore, educated agents might face two different wage levels. The price of consumption is normalized to one and subscripts are omitted for simplicity.

The budget constraint is:

$$C = (1 - L)W + B, \quad (2)$$

where  $W$  is the net wage ( $W = w(1 - t)$ ),  $t$  is the linear tax rate and  $B$  any is non-labor income.

### 2.2.1 Non-labor income

Non labor income is the sum of savings and a lump-sum transfer  $g$  from the government (if any). Agents will differ in their savings. An educated agent can only save the amount earned working when she is not at school, while an uneducated agent works the whole

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<sup>4</sup>I will derive all the analytical analysis without linking  $\alpha$  and  $\beta$  in order to obtain the highest possible degree of generality.



period. Then  $W_0$  ( $W_0h$ ) is the amount saved in the first period for uneducated (educated) agents, respectively.

Non-labor income is:

$$B_U = g + (1 + r)W_0, \quad (3)$$

for uneducated agents, and:

$$B_E = g + (1 + r)W_0h - T, \quad (4)$$

for educated agents.

Therefore  $B_U \geq B_E$ .

In Figure 1 I present an example of the three possible budget constraints, when  $w_0 < w_L < w_H$  and  $h < 1$ . The two lines corresponding to an educated agent who can enjoy full time leisure with a consumption of  $B_E$ , and the one corresponding to an uneducated agent who can enjoy full time leisure with a consumption of  $B_U$ .

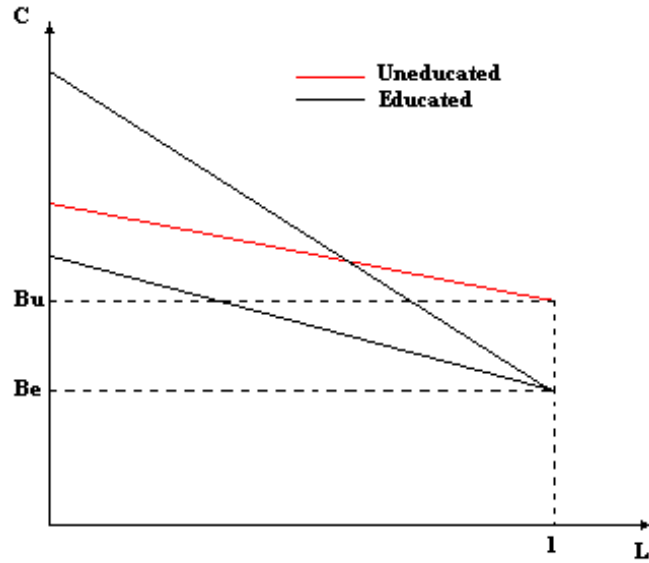


Figure 1: Budget constraints for an agent with  $h < 1$

## 2.3 Labor supply

In the second period agents choose the amount of labor and consumption to maximize their utility function.

The optimal choices are:<sup>5</sup>

$$L^* = \frac{\alpha^{1/\rho-1} W^{1/\rho-1} (W + B)}{\beta^{1/\rho-1} + \alpha^{1/\rho-1} W^{\rho/\rho-1}}, \quad (5)$$

$$C^* = \frac{\beta^{1/\rho-1} (W + B)}{\beta^{1/\rho-1} + \alpha^{1/\rho-1} W^{\rho/\rho-1}}. \quad (6)$$

To obtain an interior solution ( $1 - L^* > 0$ ) I need the following condition on wages:

$$W > \frac{\beta}{\alpha} B^{1-\rho}. \quad (7)$$

Observe that:

- i)  $B$  reduces labor supply. Then, for a given wage, an agent will work more as lower is  $B$ .
- ii) If an agent works when uneducated, she will also work if educated.
- iii) Because of (i), for a given wage, the more able among the educated individuals will work less.
- iv) Labor income is increasing in  $W$  and decreasing in  $B$ .

From the optimal choices I obtain the indirect utility function for an interior solution:

$$V(W, B) = (W + B) (\alpha\beta)^{1/\rho} \theta(W), \quad (8)$$

where  $\theta(W) = \left( \beta^{1/\rho-1} + \alpha^{1/\rho-1} W^{\rho/\rho-1} \right)^{1-\rho/\rho}$ .

In Figure 2 I present an interior optimal bundle and the utility obtained for each possible budget constraint.

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<sup>5</sup>Subscripts are omitted for simplicity

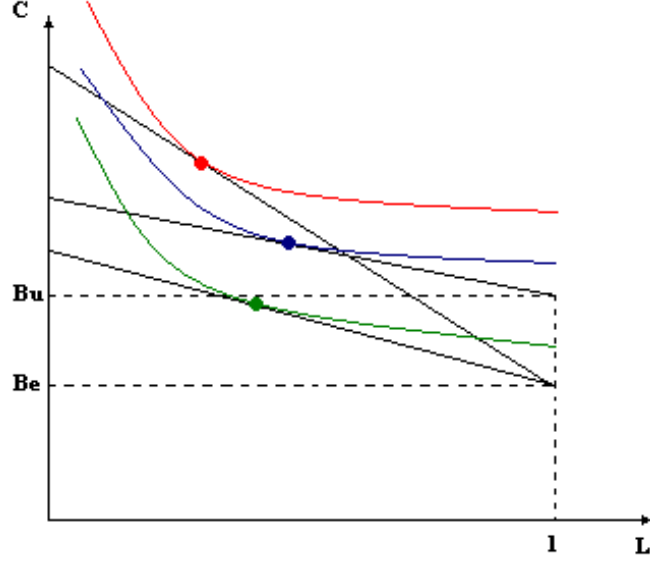


Figure 2: Leisure and Consumption choice.

## 2.4 Education decision

In the first period, anticipating their future behavior, agents decide whether to acquire education or not. An agent of ability  $h$  chooses to educate if and only if:

$$pV(W_H, B_E) + (1 - p)V(W_L, B_E) \geq V(W_U, B_U). \quad (9)$$

This also can be treated according to the expected net return to education.

$$E(g, T, h, t) = pV(W_H, B_E) + (1 - p)V(W_L, B_E) - V(W_U, B_U) \quad (10)$$

As long as the return to education is positive an agent will choose to educate. Since  $V_B > 0$ ,  $\frac{\partial B_E}{\partial h} > 0$ ,  $\frac{\partial B_E}{\partial T} < 0$  and  $\frac{\partial B_U}{\partial h} = \frac{\partial B_U}{\partial T} = 0$  it is clear that a higher ability implies a higher return to education ( $E_h > 0$ ) and a higher fee implies a lower return ( $E_T < 0$ ). Since  $V_{WB} < 0$  and  $\frac{\partial B_E}{\partial g} = \frac{\partial B_U}{\partial g} = 1$  it also easy to see that transfers have a negative effect on the return ( $E_g < 0$ ). As  $V_W > 0$  an increase in  $W_H$  or  $W_L$  increases the return to education ( $E_{W_H}, E_{W_L} > 0$ ), while an increase in  $W_U$  decreases the return to education ( $E_{W_U} < 0$ ). However, the effect of taxes is ambiguous, since an increase in taxes reduces all wages and non-labor incomes. To deal with the effect related to taxes I solve the education threshold.

Solving the inequality(9) I obtain a threshold value of ability ( $\hat{h}$ ) such that only agents with ability  $h$  in excess of  $\hat{h}$  will decide to undertake the educational training. Taking  $g$  as given, the threshold is given by:

$$\hat{h} = A_h(t) + B_h(t)g + C_h(t)T. \quad (11)$$

where  $A_h(t)$ ,  $B_h(t)$ ,  $C_h(t)$  are provided in the appendix.

From this expression I can obtain the value of  $g$  as a function of  $\hat{h}$ ,

$$g = A_g(t) - B_g(t)\hat{h} + C_g(t)T, \quad (12)$$

Again  $A_g(t)$ ,  $B_g(t)$  and  $C_g(t)$  are provided in the appendix.

## 2.5 Government

Government is assumed to have no choice variables, it will just collect taxes, pay transfers and education subsidies (if any), and fulfill revenue needs. In order to compare results I use as a benchmark a Laissez-faire case, the absence of government, that is, no taxes, no transfers, no subsidies, no revenue need.

The only imposition on the government is that it cannot incur in a deficit, that is, the sum of the revenue requirement  $R$  (if any), lump-sum transfers and education subsidies (if any) has to be less or equal than the revenue obtained. If all agents work I obtain the following inequality:

$$\begin{aligned} R + g + \xi(1 - \hat{h}) \leq & t[w_0\hat{h} + w_0 \int_{\hat{h}}^1 h dh + w_u(1 - L_u)\hat{h} + pw_H \int_{\hat{h}}^1 (1 - L_H)dh \\ & + (1 - p)w_L \int_{\hat{h}}^1 (1 - L_L)dh]. \end{aligned} \quad (13)$$

To ensure that all agents work it is enough to assume that  $W_U$  satisfies the interior solution condition, i.e. that:  $W_U > \frac{\beta}{\alpha} B^{1-\rho}$ .<sup>6</sup>

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<sup>6</sup>This condition holds in all the numerical examples.

Assuming  $R = 0$  and a uniform distribution of abilities and introducing into Equation (13) the corresponding labor supplies I obtain the following inequality:

$$g \leq \frac{D_g(t) + E_g(t)\hat{h} + F_g(t)\hat{h}^2}{G_g(t) - H_g(t)\hat{h}}. \quad (14)$$

The explicit formulas for  $C_g(t)$ ,  $D_g(t)$ ,  $E_g(t)$ ,  $F_g(t)$  and  $H_g(t)$  are provided in the appendix.

If the government returns all the revenue, Equation (14) becomes an equality. Using Equations (12) and (14) I obtain a second order equation for  $\hat{h}$ :

$$A_g(t) - B_g(t)\hat{h} + C_g(t)T = \frac{D_g(t) + E_g(t)\hat{h} + F_g(t)\hat{h}^2}{G_g(t) - H_g(t)\hat{h}}. \quad (15)$$

Rearranging, this yields:

$$\tilde{A}_g(t)\hat{h}^2 + \tilde{B}_g(t)\hat{h} + \tilde{C}_g(t) = 0, \quad (16)$$

where

$$\tilde{A}_g(t) = F_g(t) - B_g(t)H_g(t), \quad (17)$$

$$\tilde{B}_g(t) = E_g(t) + B_g(t)G_g(t) + (A_g(t) + C_g(t)T)H_g(t), \quad (18)$$

and

$$\tilde{C}_g(t) = D_g(t) - (A_g(t) + C_g(t)T)G_g(t). \quad (19)$$

Then:

$$\hat{h} = \frac{-\tilde{B}_g(t) - \sqrt{\tilde{B}_g(t)^2 - 4\tilde{A}_g(t)\tilde{C}_g(t)}}{2\tilde{A}_g(t)}. \quad (20)$$

Now, given the parameters and wages, the threshold level can be computed as a function of the lineal tax rate  $t$ .

## 2.6 Welfare

In order to perform the welfare analysis I assume a standard utilitarian welfare function with expected utilities. I assume that the government must choose the tax schedule in period one, and commits to it in the second period. The social welfare is then:

$$\begin{aligned} & \int_0^1 E[U(C(h), L(h))]dh \\ &= \int_0^{\hat{h}} V[W_U, B_U]dh + p \int_{\hat{h}}^1 V[W_H, B_E]dh + (1-p) \int_{\hat{h}}^1 V[W_L, B_E]dh. \end{aligned} \quad (21)$$

In the case of a uniform distribution of abilities this can be easily computed.

## 3 Results for a linear tax model

As exposed above, any amount of lump-sum transfer has a negative effect on education since it reduces the return to education, increasing the threshold level of ability and so less people will get higher education. As the lump-sum transfer has this negative effect, reducing the transfer the effect of the tax system on education becomes less negative. To start with, I consider the case in which the government keeps all the revenue, i.e., there is no lump-sum transfer at all ( $g = 0$ ). In that case I can rewrite the ability threshold as:

$$\begin{aligned} \hat{h} &= \frac{(1+r)W_0\theta(W_U) + W_U\theta(W_U) - pW_H\theta(W_H) - (1-p)W_L\theta(W_L)}{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]} \\ &\quad + \frac{1}{(1+r)W_o}T. \end{aligned} \quad (22)$$

Interestingly, the derivative of this first term of the function with respect to  $t$  is negative as long as consumption and leisure are complements. Then if education has no direct cost ( $T = 0$ ), as  $t$  raises the threshold level of ability decreases and more people gets higher education. This result is surprising since usually the tax schedule has a neutral effect on education in the absence of direct costs or negative in the presence of them. The presence of this direct cost easily counteracts the positive effect related to the first term, although for very low levels of cost the effect might be still positive for low tax

levels, turning negative for high tax levels. In the absence of direct costs any negative effect of  $t$  on education must be due to the existence of lump-sum transfers or other non-deductible costs because the tax rate per se increases the amount of educated agents.<sup>7</sup> Eaton and Rosen (1980b) find a similar result in a model with continuous human capital and identical agents.

The other polar case is when the government returns all the revenue. In this case, when I solve numerically, the total effect on education is always negative, so  $\hat{h}$  raises and less people gets higher education. The effect of the transfer dominates the effect of simple taxation. It is a standard result that taxation discourages investment in human capital. We have seen that in my framework taxation does not have a negative impact on education per se. It is through the existence of non deductible cost and lump-sum transfers that the tax system discourages education. The question that arises is why taxes decrease the threshold of education. The tax reduces the opportunity cost of education and the future profit of any decision. A linear tax reduces the saved amount during the first period in  $tw_0$  for uneducated and in  $tw_0h$  for educated agents. All wages are reduced by the same percentage, but agents can adjust their behavior in the second period, choosing labor and consumption, while in the first period they can only change their education decision as they supply labor in an inelastic way. This result holds even when education does not involve any kind of uncertainty and utility is derived in both periods as shown in Viiano (2007).

From an efficiency point of view, it becomes clear from Equation (5) that an increase in non-labor income (i.e. a lump-sum transfer) has a negative effect on labor supply, even when the labor supply is backward sloping in wage. Note that the tax might increase the labor supply.

To implement a tax on income to raise revenue has a negative effect on welfare, but at the same time it increases education, increasing then production and revenue. If the revenue is returned as a transfer, education decreases as also labor supply. Total effect is ambiguous.

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<sup>7</sup>A direct cost as presented, as a tuition fee, is a non-deductible cost, but other non-deductible costs (or profits) might exist, as an effort difference between education and labor.

### 3.1 Simulated results.

Due to the intractability of the mathematical expressions, the model cannot be solved analytically. In the sequel I will focus on numerical examples.

In particular, I am going to assign the following values to the parameters:

$\alpha$	$\beta$	$\rho$	$p$	$r$	$w_0$	$w_U$	$w_L$	$w_H$
0.9	0.1	-1.5	0.5	0.05	1	1.5	2	3

The values for  $\alpha$ ,  $\beta$  and  $\rho$  are similar to the ones found by Stern(1976), I set arbitrarily  $p$  to one half for simplicity. The crucial decision is how to set wages since they have a strong effect on the results. As leisure is normalized to one the wage should be set accordingly. I normalize  $w_0$  and set it equal to one.

The values assumed for the wages and the interest rate are highly relevant since they will determine the particular values obtained in the simulation (threshold level of education, labor supply, revenue, welfare...). Still the effect of taxes will go in the predicted direction independently of the particular values.

In that particular simulation, as both periods have the same length, the wages are assumed to be of similar magnitude. The interest rate is low. The concrete values presented here are merely explanatory and assume that educated agents earn between 33% and 100% more than uneducated agents, 66% more in mean. Labour supply is close to 40% of the disposable time. Most of the education thresholds are interior, being the initial value for the education threshold more or less acceptable (between one half and one quarter of population depending on the case).

In reality the length of both periods is different, being the second period three or four times larger than the first one. This implies in the model that the relation between wages must be different, the first period wage must be lower and the interest rate must be higher.

Also individuals extract utility in the first period and discount their future levels of utility. They value more current losses than future profits. As education represents an actual loss compared to a future expected profit it seems reasonable to overweight the first period wage, as it represents the utility derived from the first period, in comparison to the future gain of utility related to the second period wages.



**Example 1:** Linear tax model with only opportunity cost of education ( $T = 0$ ).

The results for this case are displayed in Figures 3 to 5

Figure 3 displays the threshold level of education as a function of  $t$  when the whole revenue is returned as a transfer. As a reference I also represent the threshold in the Laissez-faire case, which is  $\hat{h}_{lf} = A_h(0) \simeq 0.58$ . If all revenue is returned, the education threshold is monotonically increasing in  $t$  and therefore always above the Laissez-faire threshold.

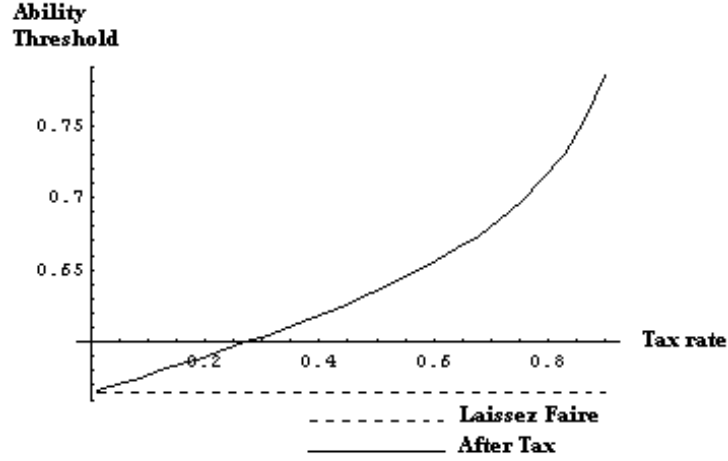


Figure 3: Linear tax returning all revenue

In Figure 4 I represent the threshold level if no revenue is returned ( $g = 0$ ). The education threshold is monotonically decreasing and therefore always below the Laissez-faire threshold.

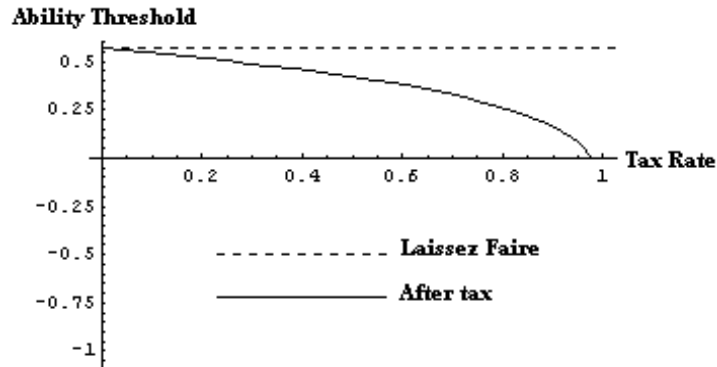


Figure 4: Ability threshold when  $g=0$

In Figure 5 I represent after-tax social welfare when all revenue is returned and, as

reference, the Laissez-faire welfare, that is 1.337. The after tax welfare is monotonically decreasing in  $t$  and therefore always below the Laissez-faire level.

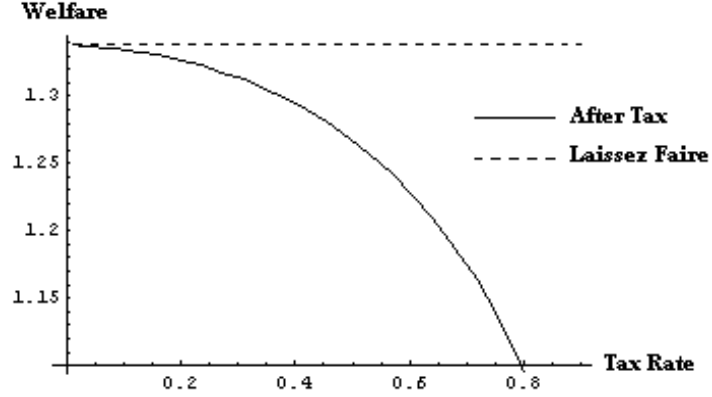


Figure 5: Social Welfare with linear tax

For any tax level, the effect on welfare is negative. Social welfare cannot be increased with a linear tax schedule above the level obtained in the Laissez-faire scenario.

Since returning all the revenue has a negative effect on education, while not returning any revenue has a positive effect, it seems that, by continuity, there must be some value of the transfer that keeps education constant.

I can find the size of the lump-sum transfer that keeps the education level fixed at the Laissez-faire level by plugging the Laissez-faire threshold in Equation (12).

$$g = A_g(t) - B_g(t)\hat{h}_{lf}. \quad (23)$$

In Figure 6 I represent the revenue and the amount of the lump-sum transfer that keeps the education fixed at the Laissez-faire level, as a function of  $t$ .

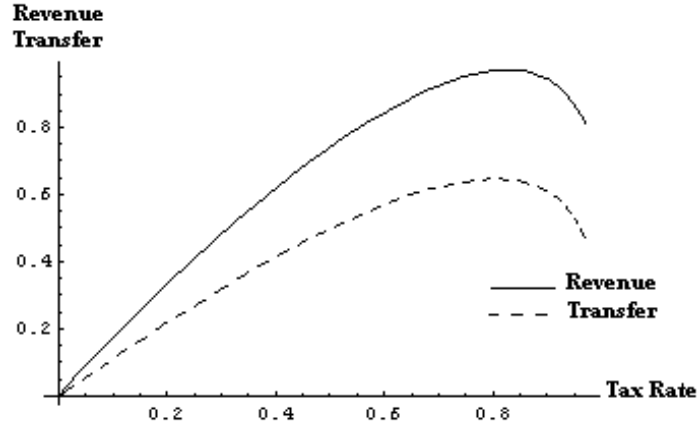


Figure 6: Revenue and lump-sum transfer

For a given tax rate, any lump-sum transfer above the level of the transfer displayed in Figure 6 will discourage education and any transfer below that level will increase education.

In Figure 7 I present after-tax welfare in this case. As expected, welfare is decreasing and it is clearly below the welfare level obtained if all revenue is returned.

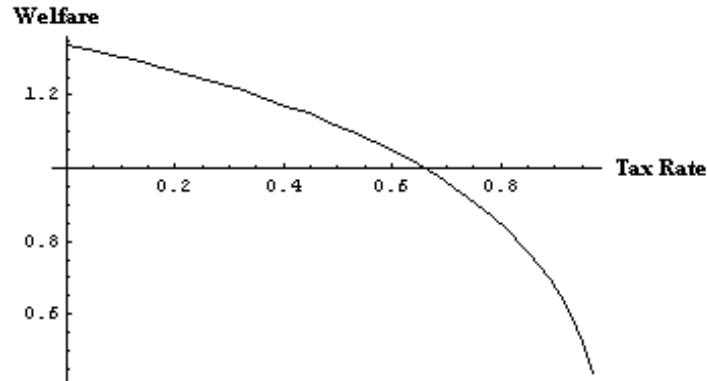


Figure 7: Welfare when education is held constant

**Example 2:** Linear tax schedule with a tuition cost: ( $T = Q = 0.2$ )

In this example I compute the model with the tuition fee using the same parameters as in Example 1. The results are shown in Figures 8 and 9.

In Figure 8 I represent the education threshold when all revenue is returned as a transfer. As expected, the threshold level is increasing in  $t$ . The inclusion of a direct cost of education has only skipped up the education threshold in the Laissez-faire case

until 0.755184. Observe that the effect of the tuition fee on the educational decision is high.

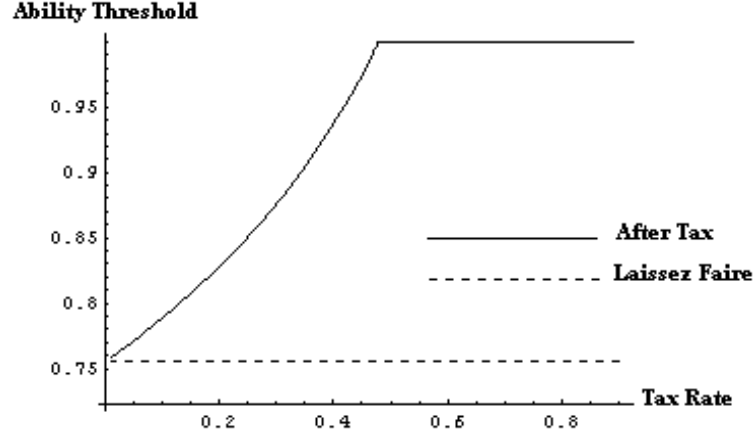


Figure 8: Ability threshold with tuition fee

Now there exist a threshold tax rate that makes education worthless to any agent, so nobody educates, even the most able, as the return of education does not compensate the monetary cost associated to it. In fact within this framing there always exist a cost of education that makes agents not willing to educate in any case.

In Figure 9 I represent jointly the Laissez-faire welfare and the welfare obtained with a linear tax schedule when revenue is returned as transfers. Welfare with taxes is decreasing in  $t$  and it is always below the level in Laissez-faire.

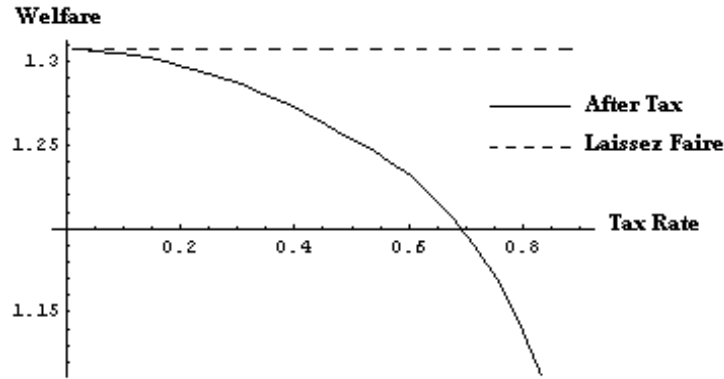


Figure 9: Welfare with tuition fee

Now I assume that the tuition cost is fully subsidized by the government ( $T = 0, \xi = Q = 0.2$ ). Educated agents do not pay any fee but a part of the revenue of the government is used to finance education.

I focus on the case in which the government returns all the remaining revenue as a lump-sum transfer. I limit the analysis in the simulation to tax levels which raise enough revenue to finance education, so I rule out any situation where  $g$  is negative.<sup>8</sup>

This will affect drastically the threshold level of education compared to the case with a tuition fee. In this case the Laissez-faire situation is the same as in the case with a tuition fee, as there is no government subsidizing the education.

In Figure 10 I represent all possible values of the lump-sum transfer/tax, the tax revenue minus the education subsidy. The result is negative for low taxes because the revenue is not high enough to subsidize all the cost of education. There is a wide range of taxes where a positive lump-sum transfer is possible. The tax rate needed to achieve a positive transfer is quite low ( $\simeq 5\%$ ). A higher tuition fee will increase the tax needed to subsidize education.

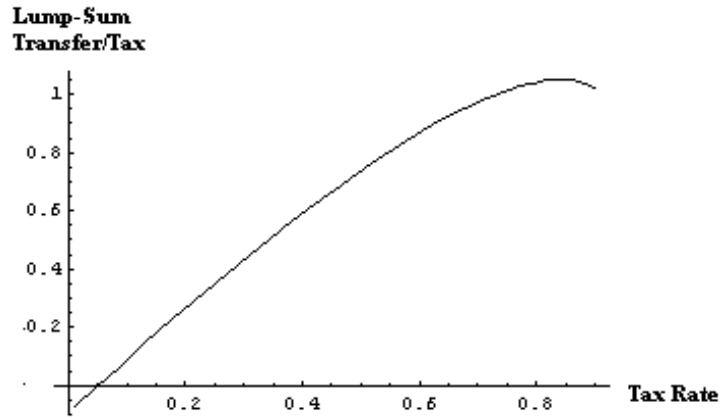


Figure 10: Lump-sum Transfer or Tax

In Figure 11 I represent the education threshold after taxation and in the Laissez-faire situation. Once again we see that as  $t$  rises the threshold level of education rises as well, due to the transfer related to taxes. However the existence of the subsidy keeps the education threshold below the corresponding Laissez-faire threshold for nearly any possible tax rate. Only very high transfers, related to very high taxes, induce threshold levels above the Laissez-faire threshold.

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<sup>8</sup>An alternative is to reduce the cost of education only by the amount collected through taxes maintaining the lump-sum transfer at zero until the revenue is high enough to cover the cost of education.

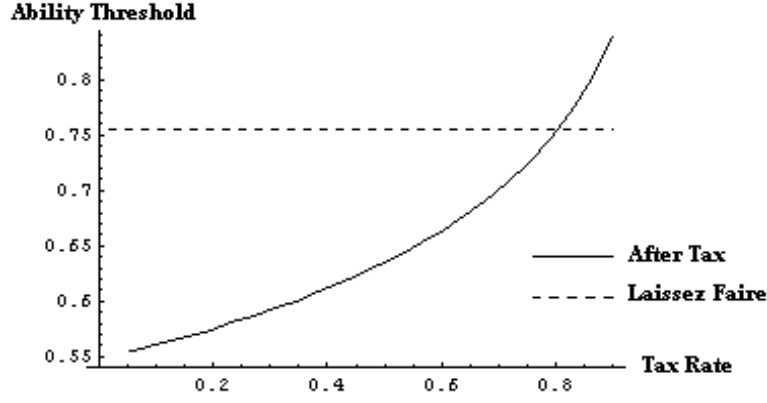


Figure 11: Ability Threshold when education is subsidized

In Figure 12 I show welfare only for tax levels with non negative transfer as well as the Laissez-faire welfare. Once again welfare is decreasing in  $t$ .

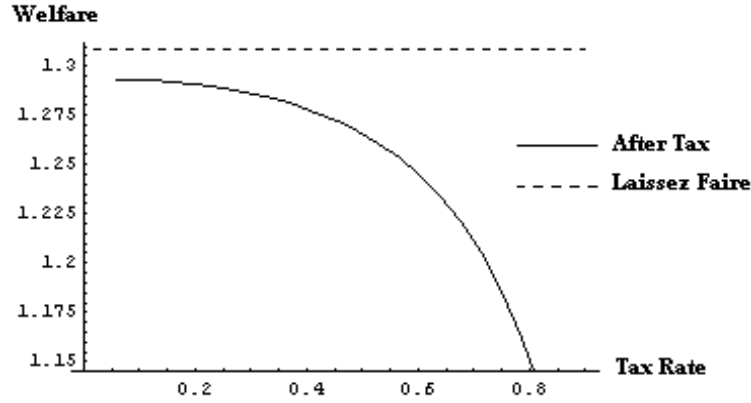


Figure 12: Welfare when education is subsidized

### 3.2 Comparing a lump-sum tax with a linear tax system

I assume now that the government has some fixed revenue need ( $R$ ) as a sunk cost. As taxes have a negative effect on welfare I assume that the tax schedule is set in such a way that only an amount enough to cover the revenue need is collected. For simplicity the tuition cost is assumed to be zero.

If the revenue is collected using a lump-sum tax, each individual must pay  $R$ , that is equal to a reduction in non-labor income. This will have effects on the education decision and so a lump-sum tax is distortionary in this framework. Although it does not affect the ratio between consumption and leisure, it will affect the education decision. In particular the threshold level of education is now:

$$\begin{aligned}\widehat{h}_1 = & \frac{(1+r)w_0\theta(w_U) + w_U\theta(w_U) - pw_H\theta(w_H) - (1-p)w_L\theta(w_L)}{(1+r)w_0[p\theta(w_H) + (1-p)\theta(w_L)]} \\ & - \frac{\theta(w_U) - p\theta(w) - (1-p)\theta(w_L)}{(1+r)w_0[p\theta(w_H) + (1-p)\theta(w_L)]}R.\end{aligned}\quad (24)$$

It is clear that there are more educated agents since  $R$  reduces the threshold of education.

If instead a linear tax is used, the tax rate must satisfy:

$$\begin{aligned}R = t[w_0\widehat{h}_2 + w_0\int_{\widehat{h}_2}^1 h dh + w_u[1 - L_U]\widehat{h}_2 + pw_H\int_{\widehat{h}_2}^1 [1 - L_H] dh \\ + (1-p)w_L\int_{\widehat{h}_2}^1 [1 - L_L] dh],\end{aligned}\quad (25)$$

where

$$\widehat{h}_2 = \frac{(1+r)W_0\theta(W_U) + W_U\theta(W_U) - pW_H\theta(W_H) - (1-p)W_L\theta(W_L)}{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]}.\quad (26)$$

I have to compute the value of the ability threshold for each  $t$  and the corresponding revenue to find the lowest value of  $t$  that makes the revenue equal to  $R$ . This can be easily done within the same framework I have used until now.

**Example 4:** Lump-sum tax versus Linear tax schedule.

In this example I assume the same parameters as in the first example and a revenue need of 0.5.<sup>9</sup>

With this data the results of the simulation show that a lump-sum tax reduces education from 0.58 to 0.49, and welfare from 1.34 to 1.14. A linear tax of 0,303 collects a revenue of 0.5, education falls to 0.51, and welfare to 1.3.

When there exist a revenue need, the use of a linear tax is less distortionary than the use of a lump-sum tax. The effects on both education and welfare are larger using a

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<sup>9</sup>The value of  $R$ , as long as it is positive, will only affect the difference between the performance of the different tax schedules, as higher is  $R$ , the higher is the difference and the better does the linear tax perform in comparison.

lump-sum tax. But the linear tax also affects the ratio between consumption and leisure. In particular the ratio raises. A similar result is also present in Eaton and Rosen (1980b). The use of a mixture of both systems remains for future research.

## 4 Conclusion.

In the absence of a direct cost, in the form of a tuition fee or similar, a linear tax rate has positive effects on education. Returning some part of the revenue as a lump-sum transfer the government can increase or decrease to some extent the threshold level of ability that makes education profitable. Nevertheless this policy always originates a social welfare loss if education does not have any other effect on society. Consequently, if the objective of the government is to maximize social welfare it should not implement a linear tax system. However if the objective is to change the education decision, a linear tax system could work. This could be the case if, for example, the level of education produces positive externalities. The government can increase social welfare if the externality related to education is high enough. If education affects the utility as a consumption good, (Alstadsæter(2001)), the whole analysis will change.

The positive effect of taxation is easily offset by the existence of a direct cost since a tuition cost has a strong negative effect on education.

In the case that the tax system is implemented to cover some revenue need, the use of a linear tax system is preferred to a lump-sum transfer if the government is a social welfare maximizer. In the case that government has some revenue need, that can be for example the cost to provide public goods, or to provide publicly some private goods, then, from a welfare point of view, a linear tax on income performs better than a lump-sum tax. It also produces less distortions in the educational decision.



## 5 Appendix

The presented indirect utility function corresponds to a case where agents choose to work, so an interior solution to the problem.

If the condition for an interior solution does not hold ( $W \leq \frac{\beta}{\alpha}B^{1-\rho}$ ), the indirect utility function must be:

$$V(W, B) = (\alpha B^\rho + \beta)^{1/\rho}. \quad (27)$$

The paper is solved for situations where all agents work.

The expressions related in the labor supply choice are

$$A_h(t) = \frac{(1+r)W_0\theta(W_U) + W_U\theta(W_U) - pW_H\theta(W_H) - (1-p)W_L\theta(W_L)}{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]}, \quad (28)$$

$$B_h(t) = \frac{\theta(W_U) - p\theta(W_H) - (1-p)\theta(W_L)}{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]}, \quad (29)$$

and

$$C_h(t) = \frac{p\theta(W_H) + (1-p)\theta(W_L)}{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]} = \frac{1}{(1+r)W_0}.$$

The expressions related to the formulation of  $g$  as a function of  $\hat{h}$  are

$$A_g(t) = \frac{(1+r)W_0\theta(W_U) + W_U\theta(W_U) - pW_H\theta(W_H) - (1-p)W_L\theta(W_L)}{p\theta(W_H) + (1-p)\theta(W_L) - \theta(W_U)}, \quad (30)$$

$$B_g(t) = \frac{(1+r)W_0[p\theta(W_H) + (1-p)\theta(W_L)]}{p\theta(W_H) + (1-p)\theta(W_L) - \theta(W_U)}, \quad (31)$$

and

$$C_g(t) = \frac{p\theta(W_H) + (1-p)\theta(W_L)}{p\theta(W_H) + (1-p)\theta(W_L) - \theta(W_U)}. \quad (32)$$

The expressions related to the government constraint are

$$\begin{aligned} D_g(t) = & \frac{1}{2}w_0 + pw_H[\Psi(W_H) - \frac{1}{2}(1+r)(1-t)w_0\Phi(W_H)] \\ & + (1-p)w_L[\Psi(W_L) - \frac{1}{2}(1+r)(1-t)w_0\Phi(W_L)] - \frac{\xi}{t}, \end{aligned} \quad (33)$$

$$\begin{aligned} E_g(t) = & w_0 + w_U[\Psi(W_U) - (1+r)(1-t)w_0\Phi(W_U)] \\ & - pw_H\Psi(W_H) - (1-p)w_L\Psi(W_L) + \frac{\xi}{t}, \end{aligned} \quad (34)$$

$$\begin{aligned} F_g(t) = & \frac{1}{2}[pw_H(1+r)(1-t)w_0\Phi(W_H) \\ & + (1-p)w_L(1+r)(1-t)w_0\Phi(W_L) - w_0], \end{aligned} \quad (35)$$

$$G_g(t) = \frac{1}{t} + pw_H\Phi(W_H) + (1-p)w_L\Phi(W_L), \quad (36)$$

and

$$H_g(t) = pw_H\Phi(W_H) + (1-p)w_L\Phi(W_L) - w_U\Phi(W_U), \quad (37)$$

where

$$\Phi(W_i) = \frac{\alpha^{1/\rho-1}W_i^{1/\rho-1}}{\beta^{1/\rho-1} + \alpha^{1/\rho-1}W_i^{\rho/\rho-1}}, \quad (38)$$

and

$$\Psi(W_i) = \frac{\beta^{1/\rho-1}}{\beta^{1/\rho-1} + \alpha^{1/\rho-1}W_i^{\rho/\rho-1}}, \quad (39)$$

for  $i = U, H, L$ .

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